From: Philip Corbin
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To: Christian Fellowship
Subject: The Golden Ratio, the Scriptures and the Cross

Greetings Brethren in the Wonderfully Wholesome Name of JESUS!!

I thought I would write an article on "The Golden Ratio, the Scriptures, and the Cross". It is going to be lengthy, but (hopefully!) educational and enlightening, so bear with me!

First, some background. The Italian Leonardo of Pisa (c.1175 – c.1240 AD), otherwise known as Leonardo Fibonacci, is generally reckoned to have been the most outstanding mathematician of the Middle Ages. Early mathematical training in Algeria, coupled with extensive travel whereby he acquired a broad knowledge of Greek and Arabic mathematics, enabled him to write his greatest work, the 'Liber Abaci', literally 'Book of the Abacus', in 1202 AD (revised 1228 AD). This book was instrumental in displacing the clumsy Roman numeration system by the Hindu-Arabic decimal system which is similar to that used today.

Fibonacci is however mainly remembered by an almost trivial recreational problem which appeared in Liber Abaci. It can be summarized as follows: - Given a new-born pair of rabbits in an enclosure, and given that any new-born pair requires one month to mature, and one month thereafter to reproduce, determine the number of pairs of rabbits in the enclosure after n months. The answer is supplied, for successive months, by the number sequence 1, 1, 2, 3, 5, 8, 13, etc. Historically, this is the earliest example in mathematics of a "recursive" series; apart from the first two terms, each term is the sum of the preceding two.

No serious study of this number sequence was undertaken until the 19^{th} century, when the French mathematician Edouard Lucas (1842-1891) examined it in depth, and dubbed it (in 1877) the Fibonacci sequence, referring to its numbers as the Fibonacci numbers. Lucas also investigated recursive series of the same type (now called generalized Fibonacci series) that begin with any two positive integers a and b (i.e. a, b, a+b, a + 2b, 2a + 3b, etc.) The Fibonacci series is the simplest such series, the next simplest (and closely related) sequence is that named after Lucas himself: 1,3 4, 7, 11 18… etc., the number of which are known as the Lucas numbers.

The Fibonacci numbers govern countless things in nature; hence they have also been referred to as "nature's numbers". For example, the number of pigs in a litter, or the number of eggs in a bird's nest, or the number of petals in most flowers, tends to be a Fibonacci number. The iris has 3 petals, the primrose 5, a delphinium 8, Marigolds and the ragwort 13, an aster 21, and various kinds of daisies 13, 21, 34, 55, or 89 petals. Romantics should note that most of these numbers are odd numbers, so that the "he/she loves me, he/she loves me not" petal picking game is best carried out starting with "he/she loves me" since that gives the best chance of ending with "he/she loves me". Four leaf clovers really are rare, because 4 is not a Fibonacci number.

Fibonacci numbers are also common re the growth of plants, the layout of seeds and in the number of spirals contained in God's design of fruit and vegetables. Re plant growth, the number of branches above the ground as the plant grows upwards typically follows Fibonacci numbers. Re spirals, the pine cone or a cauliflower typically has 5 rows of clockwise spirals, and 8 anticlockwise; the pineapple, 8 and 13; romanesque (a cross between broccoli and a cauliflower) 13 and 21. The number of spirals on a typical sunflower are usually Fibonacci numbers: 21 or 34 running clockwise, and 34 or 55 running anticlockwise respectively. Sunflowers with 55 and 89, 89 and 144, and even 144 and 233 spirals in one confirmed case, have been found. Next time you cut a fruit, count the number of sections; you will typically find a Fibonacci number – e.g. bananas have 3 sections, apples 5. The number of flat surfaces enclosing the banana will also be a Fibonacci number. Next time you are standing bored in a Budg Buy checkout queue, you now have something to do to pass the time - take out your bananas or cauliflowers etc. and start counting! You should have more fun shopping from now on!

Leaves on the stems of trees or plants are located on a spiral that winds around the stem (God designed it this way so that maximum sunlight falls on the leaves). The next time you are in your garden, go to a plant, and count the number of turns (T) of the spiral before you find a leaf directly above (or below) the leaf you started with. Also count the number of leaves (L) in that turn, including the one you started with. You should find that T and L will both be Fibonacci numbers; e.g. T/L for fruit trees is usually 2/5, plantains 3/8, leeks 5/13 etc.

A key property of any of the generalized Fibonacci sequences is that the ratio between two consecutive numbers is alternatively greater or smaller than the irrational number $(\sqrt{5} + 1)/2$, approaching it as a limit. This number has been dubbed "the **Golden Ratio**" or Phi (ϕ). It has also been called the Golden Mean or the Golden Section, or the Divine Proportion (the "Golden Ratio" appellation was given to the number in 1835 by the mathematician Martin Ohm, whose physicist brother discovered Ohm's law, one of the most basic laws in electricity). The Golden Ratio, i.e. 1.61803… , is the most "irrational" number in mathematics (a rational number can be expressed in the form of a fraction a/b where both a and b are integers).

Consider the following straight line:

————————————————————— ϕ —————————————————

The Golden ratio point (ϕ) shown divides the line into two such that the ratio of the longer bit to the shorter bit is the same as the ratio of the longer bit to the whole.

The Golden Ratio has many fascinating and beautiful mathematical qualities. For example, it is the only number which yields its reciprocal if you subtract unity from it. It is associated closely with the number 5: the diagonals of a regular five-sided figure, i.e. a pentagon, intersect each other in the Golden Ratio. It can be written neatly in terms of the number 5 as (.5^.5*.5+.5), or as $\sqrt{[(5+\sqrt{5})/(5-\sqrt{5})]}$. It can also be beautifully written as the continuous fraction 1/(1+1/(1+1/(1+…. etc. to infinity. Also, mathematicians love the neat fact that the n^{th} Fibonacci number is approximately ϕ^n divided by the square root of 5, an approximation that rapidly improves as n gets larger.

We have already seen from Fibonacci's rabbits the connection between the Fibonacci numbers and population growth - in biology once an egg is fertilized it divides and multiplies in count until it reaches a point at which the ratio of the succeeding number of cells to the previous number is the Golden Ratio ϕ.

It seems that Our Lord used the Fibonacci numbers and the Golden Ratio to design many things in nature, including, apparently, your body. Stand up and measure the distance between the ground and your navel. Let that be one unit. Your height will then be approximately the Golden Ratio. Similarly, measure the distance between your elbow and your wrist; the Golden Ratio will then roughly give the distance between your wrist and your extended fingertips. You can also try measuring the distance from your elbow to the tip of your fingers, then comparing this with the distance from your shoulder to your fingertips; you will get the Golden Ratio or a number close to it. It is the approximate ratio of the distance from the top of your head to your shoulders compared with the distance from the top of your head to your navel, and the approximate ratio of the distance from your navel to your knee compared with the distance from your navel to the bottom of your feet.

Look at your hand and measure its joints with a ruler; you will find that the ratio of your joints from your fingertips (excluding your thumbs) to your wrist typically follow the Fibonacci numbers 2, 3, 5, 8 in ratio. You have 2 hands, 3 sections on the fingers (thumbs excluded), 5 fingers per hand, and 8 fingers patterned after the Fibonacci numbers, and those numbers cited are all Fibonacci numbers also.

There is currently a beauty competition on between ladies from various departments in the Company. The Golden Ratio provides simple tests to see who is indeed the most beautiful! It is said that in the ideal human face:

1. The length of the face divided by the width is the Golden Ratio.
2. The length of the face divided by the distance between the tip of the jaw and where the eyebrows meet is the Golden Ratio.
3. The distance between where the eyebrows meet and the line of the lips divided by the length of the nose is the Golden Ratio.

There are other measurements one can use on the face involving the Golden Ratio, but those are some main ones. If you are really interested, a Dr. Stephen Marquardt has designed a computer template, based on the geometry of pentagons, which he claims works for all races in modeling the ideal beautiful face; it is available online at http://goldennumber.net/beauty.htm. If the faces of each of our contestants were matched against it, this template would (supposedly) show who is the most beautiful. I was tempted to use it to analyse the BLPC contestants but lack the full frontal pictures of their faces required for the analysis, and at any rate, being in I.S., I already know who the winner should be! (Smile). Seriously though, perhaps all beauty contests in the future should be done by computer, the winner being the one whose body stats most closely match up with the perfect proportions of the Golden Ratio.

It has also been said that the human face most closely conforms to Golden Ratio proportions when we smile, and that this is one reason you look more beautiful or handsome when you smile.

A rectangle whose sides are in the ratio 1:G is known as a Golden Rectangle. It is said that this rectangle has been used in art and architecture for centuries and that of all rectangles, it is the most pleasing to the human eye. That's debatable, but nevertheless it is the only rectangle which is self-perpetuating; if a 1:1 square is removed from such a rectangle, a Golden Rectangle still remains. Have a look at your credit cards – they are usually close to being Golden Rectangles. Coming back to your face and your smile, it has also recently been discovered that our two front teeth shown when smiling are framed within a Golden Rectangle. It is being claimed that further teeth spacings, as viewed from the front when we smile, are also, in the perfect smile, in proportion with the Golden Ratio. For further info, check http://www.goldenmeangauge.co.uk/golden.htm.

The Golden Ratio is not just confined to outer beauty. It was shown in the 1980's that the Golden Ratio governs the internal structure of the human body also re the network of the bronchi in our lungs; and probably further discoveries along those lines await us. Viruses such as the polio virus are based in their molecular design on the Golden Ratio, which enables them to replicate quickly. The design of snowflakes is said to be also based on the Golden Ratio.

Re art, it is said that the lighted area of the face in Leonardo DaVinci's Mona Lisa, the most famous painting of all time, fits exactly inside a golden rectangle. The celebrated modern artist Salvador Dali also painted "The Sacrament of the Last Supper" picture deliberately based on the Golden Ratio – the painting has the overall dimensions of a Golden Rectangle, and the room with Christ and the disciples is enclosed within a dodecahedron – the significance of this is that there are only five solids in existence (the Platonic solids) with surfaces composed of identical polygons, and of those five, two involve the golden ratio in their geometry, namely the icosahedron and its topological dual, the dodecahedron (the other Platonic solids are the cube, the octahedron and the tetrahedron).

For music lovers, it has also been speculated that the most beautiful pieces of music reach a climax at the Golden Ratio point within the piece - i.e. at 61.803… % of the way rather than midway in the piece. Furthermore, Fibonacci numbers occur in the music notes of the piano: there are 13 semitones (8 white keys, 5 black) and 8 notes in an octave; the 5^{th} and 3^{rd} notes being the basic foundation of chords, while in any given scale, the dominant note is the 5^{th} note, which is also the 8^{th} semitone. All these are Fibonacci numbers. Typical 3 note chords are based on the Fibonacci numbers also. It is said too that Stradivarius used the Golden Ratio in the design of his famous violin, and that is this is the reason for its superior tonal quality, while rooms built in Golden Ratio proportions are said to have the best acoustics.

Then there is the "Golden Spiral". If you extend a golden rectangle into a square, then extend the square into a golden rectangle, continue ad infinitum, and then join up the points on the diagonals of all the squares in a arc, you get the so-called "Golden Spiral", a curve found all over nature. Seashells of most mollusks, e.g. snail shells and the Nautilus seashell, follow the golden spiral. So do the arms of galaxies, and the horns of antelopes, goats and rams.

If you are wondering why I am so familiar with this subject, I should mention that I studied the Fibonacci numbers and the Golden Ratio extensively while at University doing my Master's degree in Electrical Engineering. I rediscovered that the numbers govern the electrical behaviour of a class of electrical networks known as ladder networks (I say "rediscovered" because I subsequently discovered that an American, Morgan Voyce, had beaten me to the discovery in the late 1950's.). For example, the electrical impedance, in terms of Ohms, of an infinitely long ladder network composed solely of unity Ohm resistances is the Golden ratio. I also discovered and documented, for my Master's Thesis (entitled "The use of number patterns in the modeling of transmission lines and electrical fields") how the Fibonacci and Lucas numbers can be used to accurately model the electrical behaviour of high voltage transmission lines and certain electrical fields (and to my knowledge those discoveries were original).

In 1963, the Fibonacci Association was established in order to study the Fibonacci numbers and related number patterns. Based at the University of Santa Clara, California, U.S.A., its publications, notably that of the Fibonacci Quarterly, have served to maintain a high interest in the subject - in the course of my research, I studied all the issues of this journal - if you are really interested in the Fibonacci numbers, this is the publication to read; it is available at our UWI Cave Hill Campus.

We come, finally, to the Golden Ratio and the scriptures. There are two numbers which crop up often in scripture, namely 7 and 12, which are said to represent perfection and completeness respectively. In Revelation chapters 14 and 21, the numbers 144,000 and 144 appear. Notice that the 12^{th} number in the Fibonacci series is 12 squared or 144 (the numbers run 1,1,2,3,5,8,13,21,34,55,89,**144**…). Notice also that the sum of the digits of 144 is 9. In fact, only every 12^{th} Fibonacci number sums to 9. Further, for the Fibonacci numbers, or any generalised Fibonacci sequence beginning with integers a, b, a+b, etc., starting from the first number, each number in each set of 12 numbers, when added to the corresponding number in each of the next set of 12 numbers, and the digits summed, produces the number 9. In other words, the only repeating pattern within such a sequence occurs every 12 numbers, and it will be recalled (from earlier in this write-up) that the ratio of successive numbers in such a sequence is the Golden Ratio. This is the hidden, hardly known, and extremely beautiful, connection between 12 and the Golden Ratio.

Before continuing, I wish to introduce the concept (my own) of "**Fibonacci Rectangles**". These are rectangles whose height and width match successive Fibonacci numbers. By this definition, the most basic Fibonacci Rectangle (based on the first two Fibonacci numbers 1,1) is a square, and thereafter Fibonacci Rectangles are close to being Golden Rectangles.

Fibonacci Rectangles occur in both nature and the scriptures. For example, in nature, the DNA molecule, the basis of life, consists of 2 intertwined helixes; the length of the curve in each helix is 34 Angstroms and the width is 21 Angstroms (an angstrom is one hundred millionth of a centimeters); 21 and 34 are successive Fibonacci numbers, and so each DNA curve fits inside a Fibonacci Rectangle.

The Ark of the Covenant, the holiest object in the Old Testament, has the dimensions of two Fibonacci Rectangles. Scripture gives it as having a square cross section of 3/2 cubits with a length of 5/2 cubits (Exodus 25:10). Viewed from the front, it had (3,5) ratio dimensions. I wonder if it is coincidence that God specified to Moses its dimensions based on the successive Fibonacci numbers 2, 3 and 5.

The altar of burnt offering also had the dimensions of two Fibonacci Rectangles – it was 5 cubits long and wide, and 3 cubits high (Exodus 28:1).

Consider also Noah's ark. Its cross-section (30 cubits by 50 cubits) was a Fibonacci Rectangle.

Finally, and most interestingly, the dimensions of the cross or crucifix, the universal symbol of Christianity, is the most aesthetic, in my opinion, if it fits inside a Fibonacci Rectangle. Here the occurrence of the numbers 7 and 12 should be noted. Using 7 squares, a cross 3 units wide and 5 units long can be constructed, whose width and height fits inside a Fibonacci rectangle. On my website, in the Praise Pearls section, I have constructed a cross out of 12 squares, it is 5 squares wide and 8 squares long, both Fibonacci numbers, and the cross again fits inside a Fibonacci Rectangle. In the Promise Pearls section, I have constructed another cross which again fits inside a Fibonacci Rectangle of 21 by 34 units (to be honest though, this was actually unplanned on my part – the numbers automatically arose out of the design I chose, but I was extremely pleased when I discovered this!).

In the Tabernacle of Moses (Exodus 26-28), the colours blue, purple, and scarlet predominated. Of interest here is the fact that purple divides the colours blue and scarlet in the golden ratio. In general, re any 3 colours, the most aesthetically beautiful colour combinations occur when the colours are spread out along the wavelengths of colour in the visible spectrum using the Golden Ratio. I was initially ignorant of this fact when I first designed the High Priest's breastplate for my website. I used a scarlet colour which contained insufficient red. I noticed this deficiency, and so originally enclosed the whole in a red border to compensate, even though I knew the colour red was not in scripture. Only after the website was completed did the penny finally drop – that scarlet in scripture was, or close to, the colour of blood. I have now redesigned the breastplate with the true colours and the aesthetic need for red has vanished.

In passing, another interesting thing I discovered recently, from researching Jewish literature on the Internet, is that traditionally the Breastplate was designed without an equal mixture of colours as I had assumed. The ratios of blue, purple and scarlet is the same, but there was, according to tradition, a 6:1 ratio re those colours and the gold. I discovered this independently in AutoCAD when playing around with the colours in designing the breastplate for my website. I found that the breastplate colours only made visual sense aesthetically if the blue, purple and scarlet colours dominated. (I have tried to show this also in my revised design of the breastplate, as well as give a more realistic idea of the size of the breastplate gems in the breastplate, which was about 9 inches square).

Well there is more I could share but I guess that is enough info for the time being. I hope you found some, or all, of the above interesting, whether or not you are mathematically inclined.

May the Lord bless you all richly!

PC

PS… I checked the dimensions of my business card – and it is **exactly 54** mm by **89** mm… a Fibonacci rectangle!